# Section 8.1 Solutions

math 31

2. $y = \sqrt{2-x^2}$, $0 \le x \le 1$

$y' = \dfrac{-x}{\sqrt{2-x^2}}$, and

$$L = \int_0^1 \sqrt{1+(y')^2}\,dx = \int_0^1 \sqrt{1+\frac{x^2}{2-x^2}}\,dx$$

$$= \int_0^1 \sqrt{\frac{(2-x^2)+x^2}{2-x^2}}\,dx = \int_0^1 \sqrt{\frac{2}{2-x^2}}\,dx$$

$$= \sqrt{2}\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx = \sqrt{2}\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\Big|_0^1$$

$$= \sqrt{2}\left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}(0)\right) = \sqrt{2}\cdot\frac{\pi}{4} = \boxed{\frac{\pi}{2\sqrt{2}}}$$

10. $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}$, $1 \le y \le 2$

$x' = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$, and

$$L = \int_1^2 \sqrt{1+(x')^2}\,dy = \int_1^2 \sqrt{1+\left(\tfrac{1}{4}y^6 - \tfrac{1}{2} + \tfrac{1}{4}y^{-6}\right)}\,dy$$

$$= \int_1^2 \sqrt{\tfrac{1}{4}y^6 + \tfrac{1}{2} + \tfrac{1}{4}y^{-6}}\,dy = \int_1^2 \sqrt{\left(\tfrac{1}{2}y^3 + \tfrac{1}{2}y^{-3}\right)^2}\,dy$$

$$= \int_1^2 \left(\tfrac{1}{2}y^3 + \tfrac{1}{2}y^{-3}\right)dy = \left[\frac{y^4}{8} - \frac{1}{4y^2}\right|_1^2$$

$$= \boxed{33/16}$$

12. $y = \ln(\cos\theta)$, $0 \le \theta \le \frac{\pi}{3}$

$y' = \dfrac{-\sin\theta}{\cos\theta} = -\tan\theta$, and

$$L = \int_0^{\pi/3} \sqrt{1+(y')^2}\,d\theta = \int_0^{\pi/3} \sqrt{1+\tan^2\theta}\,d\theta$$

$$= \int_0^{\pi/3} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big|_0^{\pi/3}$$

$$= \boxed{\ln|2+\sqrt{3}|}$$